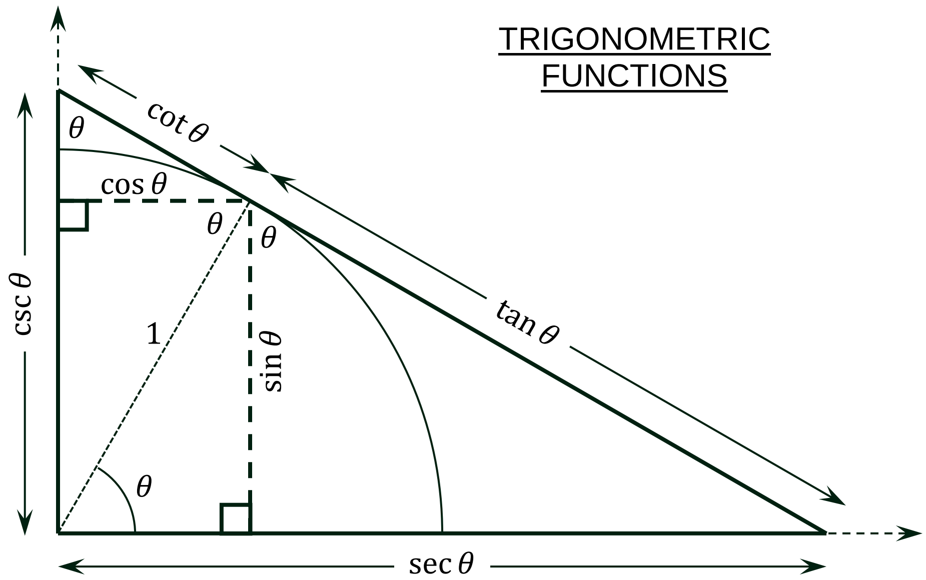
TRIGONOMETRIC FUNCTIONS



$$\sin(\theta) = \frac{O}{H}$$
$$\cos(\theta) = \frac{A}{H}$$
$$\tan(\theta) = \frac{O}{A}$$
$$\cot(\theta) = \frac{A}{O}$$
$$\sec(\theta) = \frac{H}{A}$$
$$\csc(\theta) = \frac{H}{O}$$

RECIPROCAL IDENTITIES

$\sin(\theta) = \frac{O}{H}$	$\csc(\theta) = \frac{H}{O}$	$\csc(\theta) = \frac{1}{\sin(\theta)}$
$\cos(\theta) = \frac{A}{H}$	$\sec(\theta) = \frac{H}{A}$	$\sec(\theta) = \frac{1}{\cos(\theta)}$
$\tan(\theta) = \frac{O}{A}$	$\cot(\theta) = \frac{A}{O}$	$\cot(\theta) = \frac{1}{\tan(\theta)}$

RATIO IDENTITIES

$\sin(\theta) = \frac{O}{H}$	$\csc(\theta) = \frac{H}{O}$
$\cos(\theta) = \frac{A}{H}$	$\sec(\theta) = \frac{H}{A}$
$\tan(\theta) = \frac{O}{A}$	$\cot(\theta) = \frac{A}{O}$
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$
$\tan(\theta) = \frac{\sec(\theta)}{\csc(\theta)}$	$\cot(\theta) = \frac{\csc(\theta)}{\sec(\theta)}$

PYTHAGOREAN IDENTITIES

$$\cos(\theta) = \frac{A}{H}$$

$$\sin(\theta) = \frac{O}{H}$$

$$\sin^2(\theta) + \sin^2(\theta) = 1$$

$$\frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} = \frac{1}{\cos^2(\theta)}$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$\frac{\sin^2(\theta)}{\sin^2(\theta)} + \frac{\cos^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)}$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

SYMMETRY IDENTITIES

The graph of $\sin(\theta)$ is symmetric about the origin, therefore…

$$\sin(-\theta) = -\sin(\theta)$$

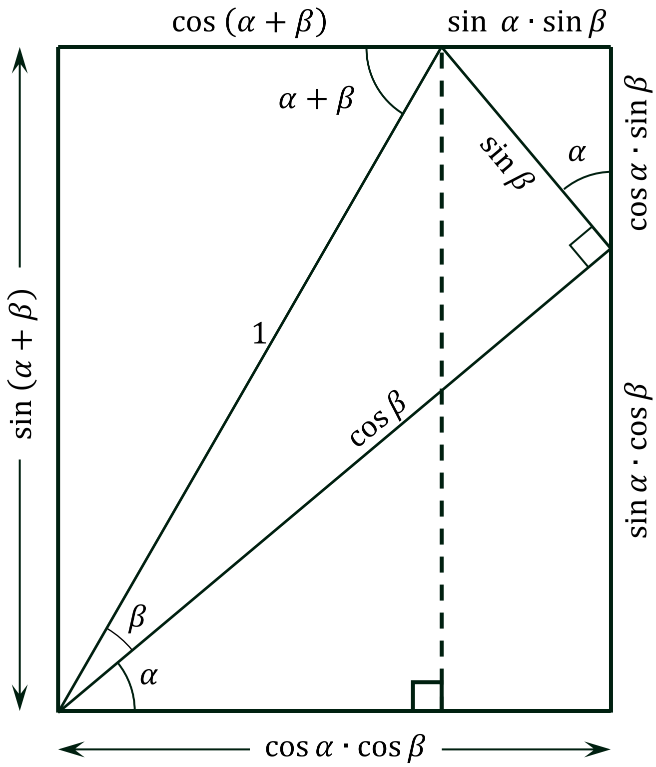
The graph of $\cos(\theta)$ is symmetric about the y-axis, therefore…

$$\cos(-\theta) = \cos(\theta)$$

The graph of $\tan(\theta)$ is symmetric about the origin, therefore…

$$\tan(-\theta) = -\tan(\theta)$$

SIN and COS of SUMS (and DIFFERENCES)



$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

$$\sin(\alpha-\beta) = \sin\big(\alpha+(-\beta)\big) = \sin\alpha\cdot\cos(-\beta) + \cos\alpha\cdot\sin(-\beta)$$

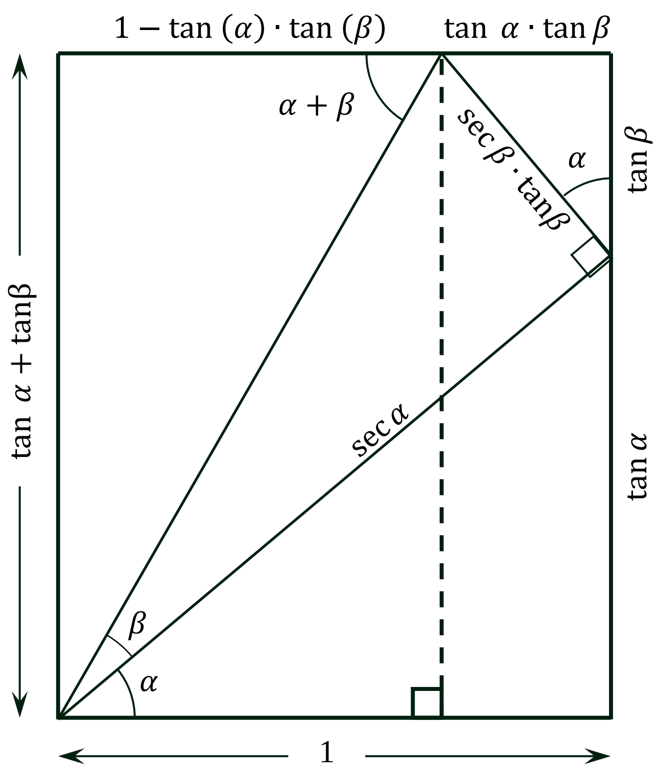
$$\sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta - \cos\alpha\cdot\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$\cos(\alpha-\beta) = \cos\big(\alpha+(-\beta)\big) = \cos\alpha\cdot\cos(-\beta) - \sin\alpha\cdot\sin(-\beta)$$

$$\cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta$$

TAN of SUMS (and DIFFERENCES)



$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}$$

$$\tan(-\beta) = -\tan\beta$$

$$\tan(\alpha-\beta) = \tan\big(\alpha+(-\beta)\big) = \frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\cdot\tan(-\beta)}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

DOUBLE ANGLE FORMULAE

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$$

$$\sin(2\theta) = \sin(\theta+\theta) = \sin\theta\cdot\cos\theta + \cos\theta\cdot\sin\theta$$

$$\boxed{\sin(2\theta) = 2\cdot\sin\theta\cdot\cos\theta}$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$\cos(2\theta) = \cos(\theta+\theta) = \cos\theta\cdot\cos\theta - \sin\theta\cdot\sin\theta$$

$$\boxed{\cos(2\theta) = \cos^2\theta - \sin^2\theta}$$

$$\cos(2\theta) = (1-\sin^2\theta) - \sin^2\theta \quad \boxed{= 1 - 2\cdot\sin^2\theta}$$

$$\cos(2\theta) = \cos^2\theta - (1-\cos^2\theta) \quad \boxed{= 2\cdot\cos^2\theta - 1}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$\tan(2\theta) = \tan(\theta+\theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta\cdot\tan\theta}$$

$$\boxed{\tan(2\theta) = \frac{2\cdot\tan\theta}{1 - \tan^2\theta}} \qquad \cdot\left(\frac{\cos^2\theta}{\cos^2\theta}\right) =$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} \qquad \boxed{= \frac{2\cdot\sin\theta\cdot\cos\theta}{\cos^2\theta - \sin^2\theta}}$$

HALF ANGLE FORMULAE

$$\cos(2\theta) = 1 - 2 \cdot \sin^2\theta$$

$$2 \cdot \sin^2\theta = 1 - \cos(2\theta)$$

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

$$\sin\theta = \pm\sqrt{\frac{1 - \cos(2\theta)}{2}}$$

$$\cos(2\theta) = 2 \cdot \cos^2\theta - 1$$

$$2 \cdot \cos^2\theta = 1 + \cos(2\theta)$$

$$\cos^2\theta = \frac{1 + \cos(2\theta)}{2}$$

$$\cos\theta = \pm\sqrt{\frac{1 + \cos(2\theta)}{2}}$$

$$\frac{\emptyset}{2} = \theta$$

$$\boxed{\sin\frac{\emptyset}{2} = \pm\sqrt{\frac{1 - \cos(\emptyset)}{2}}}$$

$$\boxed{\cos\frac{\emptyset}{2} = \pm\sqrt{\frac{1 + \cos(\emptyset)}{2}}}$$

$$\tan\frac{\emptyset}{2} = \frac{\sin\frac{\emptyset}{2}}{\cos\frac{\emptyset}{2}} = \frac{\pm\sqrt{\frac{1 - \cos(\emptyset)}{2}}}{\pm\sqrt{\frac{1 + \cos(\emptyset)}{2}}}$$

$$\tan\frac{\emptyset}{2} = \frac{\sqrt{1 - \cos(\emptyset)}}{\sqrt{1 + \cos(\emptyset)}} \cdot \frac{\sqrt{1 + \cos(\emptyset)}}{\sqrt{1 + \cos(\emptyset)}} = \frac{\sqrt{1 - \cos^2(\emptyset)}}{1 + \cos(\emptyset)} \boxed{= \frac{\sin(\emptyset)}{1 + \cos(\emptyset)}}$$

$$\tan\frac{\emptyset}{2} = \frac{\sqrt{1 - \cos(\emptyset)}}{\sqrt{1 + \cos(\emptyset)}} \cdot \frac{\sqrt{1 - \cos(\emptyset)}}{\sqrt{1 - \cos(\emptyset)}} = \frac{1 - \cos(\emptyset)}{\sqrt{1 - \cos^2(\emptyset)}} \boxed{= \frac{1 - \cos(\emptyset)}{\sin(\emptyset)}}$$

$$\boxed{\tan\frac{\emptyset}{2} = \pm\sqrt{\frac{1 - \cos(\emptyset)}{1 + \cos(\emptyset)}}}$$