DEFINITION

Geometry is a branch of mathematics that deals with the measurement, properties, and relationships of points, lines, angles, surfaces, and solids. The word geometry is derived from the words “geo”, meaning earth, and “metro”, meaning measure.

HISTORY

The Egyptians first used geometry about 2500 BC because the seasonal overflowing of the Nile made it necessary to reestablish boundaries so that taxes could be levied and collected. About 500 BC the Greeks began to develop information received from the Egyptians into the branch of mathematics we now know as geometry. By the 4th century AD, they had developed arithmetic and geometry into separate branches of mathematical science.

POINTS and LINES

Points and lines are undefined elements of geometry, yet everyone has some understanding of these terms.

- point – understood to have no length, width or thickness, but indicates a location.
- line – considered to have length, but no width or thickness.
- straight line – the shortest distance between two points.
  Sometimes a line is said to be straight if it does not curve.
- curved line – a line no part of which is straight.
- transversal – a line that cuts across two or more lines.
- parallel lines – two straight lines (in the same plane) that do not intersect.

A point is usually shown on paper as a small dot or cross and is named with a capital letter. A straight line is usually designated by the two points it connects.

Example: point “A” might be designated as… · A …or… + A
Example: line “AB” is the line between points “A” and “B”

ANGLES

There are many definitions of an angle…

- In geometry, an angle may be defined as the space between two lines diverging from a common point; the point is called the vertex.
- In trigonometry, an angle may be defined as the amount of rotation required to bring one line into coincidence with another.
- In surveying, an angle may be defined as the difference in direction of two intersecting lines.

MEASURE of an ANGLE

- degree – the most common measure of an angle is defined as 1 / 360th of a complete circle, angle or turn. We usually think of a circle as divided into 360 equal arcs. If radii are drawn to each end of one of these small arcs, the angle formed by the two radii measures one degree.
- minute – each degree is divided into 60 equal parts, with each part measuring one minute, therefore, one minute is 1 / 60th of a degree.
- second – each minute is divided into 60 equal parts, with each part measuring one second, therefore, one second is 1 / 3600th of a degree.

The symbols used for degrees, minutes, and seconds are (°), ('), and (").

Example: 36 degrees, 24 minutes, 52 seconds is… 36° 24' 52"
This same angle might be shown as 36-24-52 which is a shorthand notation sometimes used in field books and quick calculations.

The protractor can be used to measure angles on paper.
The transit and theodolite measure angles in the field.

ANGLES (continued…)

- acute angle – an angle of less than 90°.
- right angle – an angle of 90°.
- obtuse angle – an angle of more than 90° and less than 180°.
- straight angle – an angle of 180°.
- complementary – two angles are said to be complementary if their sum is 90°.
- supplementary – two angles are said to be supplementary if their sum is 180°.
- opposite vertex angles – angles opposite each other at the intersection of two crossing straight lines are equal.
- alternate interior angles – angles on opposite sides of a transversal and between two parallel lines cut by the transversal are equal.
- alternate exterior angles – angles on opposite sides of a transversal and outside of two parallel lines cut by the transversal are equal.

POLYGONS

- polygon – a closed figure bounded by straight lines lying in the same plane.
- The sum of the interior angles of a closed polygon is equal to: (n–2)180°, where n is the number of sides. Thus, the sum of the interior angles for figures with 3 sides = 180°; 4 sides = 360°; 5 sides = 540°, etc.
- The sum of the exterior angles of a closed polygon is equal to: (n+2)180°, where n is the number of sides. Thus, the sum of the exterior angles for figures with 3 sides = 900°; 4 sides = 1080°; 5 sides = 1260°, etc.

TRIANGLES

- triangle – a polygon of three sides.
- right triangle – a triangle with one right angle.
- scalene triangle – a triangle with no right angle and no two sides equal.
- isosceles triangle – a triangle with two equal sides and two equal angles.
- equilateral triangle – a triangle with three equal sides and three equal angles.

QUADRILATERALS

- quadrilateral – a four-sided polygon.
- trapezium – a quadrilateral with no equal angles and no equal sides.
- trapezoid – a quadrilateral with two opposite parallel sides.
- parallelogram – a quadrilateral with two pairs of opposite parallel sides.
- rhombus – a parallelogram in which all sides are equal.
- rectangle – a parallelogram in which all angles are right angles.
- square – a rectangle in which all sides are equal.

MULTI-SIDED POLYGONS

- 5-sided polygons are pentagons.
- 6-sided polygons are hexagons.
- 7-sided polygons are heptagons.
- 8-sided polygons are octagons.
- 9-sided polygons are nonagons.
- 10-sided polygons are decagons.
- 12-sided polygons are dodecagons.

REGULAR, CONGRUENT, and SIMILAR

- regular polygon – all angles are equal and all sides are equal.
- congruent polygons – two polygons with equal corresponding angles and equal corresponding sides.
- similar polygons – two polygons with equal corresponding angles, but with proportional corresponding sides.

CIRCLES

- circle – a closed plane curve, with all points equidistant from a point within called the center or radius point.
- radius – the distance from the center of a circle to any point on the circle.
- diameter – the distance across a circle through the center.
  One diameter is two radii.
- circumference – the distance around the perimeter of a circle.
- chord – a straight line between two points on a circle.
- secant – a straight line that intersects a circle at two points.
- tangent – a straight line that touches a circle at only one point.
- arc – any part of a circle.
- semicircle – an arc equal to one-half the circumference of a circle.
- central angle – an angle formed by two radii. The Greek letter Δ (delta) is often used to denote a central angle. A central angle has the same number of degrees as the arc it intercepts. Thus a central angle is measured by its intercepted arc.
- sector – a figure bounded by an arc of a circle and two radii of the circle.
- segment – a figure bounded by a chord and an arc of a circle.
- concentric circles – two circles with the same center, but with different radius.
- eccentric circles – two circles with different centers and with different radius.

CIRCLE and LINE RELATIONSHIPS

- The radius of a circle is perpendicular to a tangent to the circle at the point of tangency.

- The perpendicular bisector of a chord passes through the center of the circle.

- Tangents to a circle from an outside point are equal.

- A line from the center of a circle to an outside point bisects the angle between the tangents from the point to the circle.

ADDING TWO ANGLES

- When the number of minutes in a sum is sixty or more, sixty minutes are subtracted from the minutes column and one degree is added to the degree column. Likewise, when the number of seconds in a sum is sixty or more, sixty seconds are subtracted from the seconds column and one minute is added to the minutes column.

  Example: to add 32° 46' 32" and 14° 22' 44"…

write…		32° 46' 32"				
	+	14° 22' 44"				
and add…		46° 68' 76"	=	46° 69' 16"	=	47° 09' 16"

ADDING MULTIPLE ANGLES

- When a number of angle measurements are added, as is common in surveying, each column (degrees, minutes and seconds) is added separately and recorded. If the sum of either the minutes column or the seconds column, or both, is sixty or more, the same procedure is followed as in adding two angles.

  Example: to add the following angle measurements…

write…		93° 18' 22"				
		65° 13' 08"				
		218° 19' 30"				
		67° 05' 20"				
	+	96° 04' 50"				
and add…		539° 59' 130"	=	539° 61' 10"	=	540° 01' 10"

SUBTRACTING TWO ANGLES

- When subtraction of seconds would result in a negative number, one minute is subtracted from the minutes column and sixty seconds are added to the seconds column. Likewise, when subtraction of minutes would result in a negative number, one degree is subtracted from the degree column and sixty minutes are added to the minutes column.

  Example: to subtract 52° 33' 50" from 96° 08' 14"…

write…		96° 08' 14"	=	95° 67' 74"
	–	52° 33' 50"	=	52° 33' 50"
and subtract…				43° 34' 24"

AVERAGE of MULTIPLE ANGLES

Surveyors often measure angles by repetition. An angle of 36° 30' 30" might have been read on the first reading as 36° 30', but after turning the angle six times, the accumulated angle may have read 216° 03' 00". Dividing this by six yields 36° 30' 30", which is closer to the true measurement.

- The degrees of an accumulated angle are reduced to multiples of the divisor and any remainder is multiplied by sixty and added to the minutes.
- The minutes, after any additions, are then reduced to multiples of the divisor and any remainder is multiplied by sixty and added to the seconds.
- The seconds, after any additions, are not modified any further.
- Divide the resulting degrees, minutes, and seconds – round the decimal seconds appropriately.

Example:	to average 347° 08' 02" as accumulated from six angles…		
write…	(347° 08' 02") ÷ 6	=	
	(342° 308' 02") ÷ 6	=	
	(342° 306' 122") ÷ 6	=	57° 51' 20.33"

CHANGING "DEGREES, MINUTES, SECONDS" to "DECIMAL DEGREES"

- First, divide the seconds by sixty and add the quotient to the minutes – the result is minutes and decimals of a minute.
- Then, divide the minutes and decimals of a minute by sixty and add the quotient to the degrees – round the decimal degrees appropriately.

Example:	to change 46° 24' 36" to decimal degrees…	
write…		46° 24' 36"
	36" ÷ 60 = 0.6000'	46° 24.6000'
	24.6000' ÷ 60 = 0.4100°	46.4100°

CHANGING "DECIMAL DEGREES" to "DEGREES, MINUTES, SECONDS"

- First, keeping the integer part of the degrees separate, multiply the decimal fraction of the degrees by sixty – the resulting product is in minutes and decimals of a minute.
- Then, keeping the integer part of the minutes separate, multiply the decimal fraction of the minutes by sixty – the resulting product is in seconds and decimals of a second – round the decimal seconds appropriately.
- Combine the integer degrees, integer minutes, and resulting seconds into the final answer.

Example:	to change 36.12345° to degrees, minutes, and seconds…	
write…		36.12345°
	0.12345° x 60 = 7.407'	36° 07.407'
	0.407' x 60 = 24.42"	36° 07' 24.42"