TERRAMETRA

LINEAR EQUATIONS

Terrametra Resources

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1.1 EQUATIONS

- Basic Terminology
- Linear Equations
- Identities, Conditional Equations, and Contradictions
- Solving for a Specified Variable (Literal Equations)

BASIC TERMINOLOGY

An ***equation*** is a statement that two expressions are *equal*.

$x + 2 = 9$ $\quad 11x = 5x + 6x$ $\quad x^2 - 2x - 1 = 0$

To solve an equation means to find all numbers that make the equation a true statement. These numbers are the ***solutions***, or ***roots***, of the equation. A number that is a solution of an equation is said to *satisfy* the equation, and the solutions of an equation make up its ***solution set***.

Equations with the same solution set are ***equivalent equations***.

Let $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ represent real numbers.

If $\boldsymbol{a} = \boldsymbol{b}$, then $\boldsymbol{a} + \boldsymbol{c} = \boldsymbol{b} + \boldsymbol{c}$.

The same number may be added to each side of an equation without changing the solution set.

Let $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ represent real numbers.

If $\boldsymbol{a} = \boldsymbol{b}$ and $\boldsymbol{c} \neq \mathbf{0}$, then $\boldsymbol{ac} = \boldsymbol{bc}$.

Each side of an equation may be multiplied by the same nonzero number without changing the solution set.

LINEAR EQUATIONS

(First Degree Equations)

A ***linear equation*** in one variable is an equation that can be written in the form

$$\boldsymbol{ax + b = 0}$$

Where $\boldsymbol{a}$ and $\boldsymbol{b}$ are real numbers with $\boldsymbol{a \neq 0}$.

Linear equations	Nonlinear equations
$3x + \sqrt{2} = 0$	$\sqrt{x} + 2 = 5$
$\frac{3}{4}x = 12$	$\frac{1}{x} = -8$
$0.5(x + 3) = 2x - 6$	$x^2 + 3x + 0.2 = 0$

A linear equation is also called a ***first-degree equation*** since the greatest degree of the variable is 1.

Example 1

Solving a Linear Equation

1(a) Solve: $3(2x-4)=7-(x+5)$

Be careful with signs.

Solution:

$6x-12=7-x-5$ Distributive property.

$6x-12=2-x$ Combine like terms.

$6x-12+x=2-x+x$ Add x *(both sides)*.

$7x-12=2$ Combine like terms.

$7x-12+12=2+12$ Add **12** *(both sides)*.

$7x=14$ Combine like terms.

$$\frac{7x}{7}=\frac{14}{7}$$

Divide by **7** *(both sides)*.

$x=2$

Example 1

Checking the Solution

Check:

Checking the solution is recommended.

$$3(2x - 4) = 7 - (x + 5)$$ Original equation.

$$3(2(2) - 4) \stackrel{?}{=} 7 - ((2) + 5)$$ Let $x = 2$.

$$3(4 - 4) \stackrel{?}{=} 7 - (7)$$ Simplify.

$$0 = 0$$ True.

The solution set is $\{2\}$.

Example 2

Solving a Linear Equation with Fractions

2(a) Solve: $$\frac{2x+4}{3}+\frac{1}{2}x=\frac{1}{4}x-\frac{7}{3}$$

Solution: Multiply by **12** (the *LCD* of the fractions).
Distribute the **12** to ***all*** terms within parentheses.

$$12\left(\frac{2x+4}{3}+\frac{1}{2}x\right)=12\left(\frac{1}{4}x-\frac{7}{3}\right)$$

$$12\left(\frac{2x+4}{3}\right)+12\left(\frac{1}{2}x\right)=12\left(\frac{1}{4}x\right)-12\left(\frac{7}{3}\right)$$

$$4(2x+4)+6x=3x-28$$ Multiply.

Example 2

Solving a Linear Equation with Fractions

Solution (cont'd):

$$4(2x + 4) + 6x = 3x - 28$$

$$8x + 16 + 6x = 3x - 28$$ Distributive property.

$$14x + 16 = 3x - 28$$ Combine like terms.

$$11x + 16 = -28$$ Subtract $\mathbf{3x}$ *(both sides)*.

$$11x = -44$$ Subtract $\mathbf{16}$ *(both sides)*.

$$x = -4$$ Divide by $\mathbf{11}$ *(both sides)*.

Example 2

Checking the Solution

Check:

$$\frac{2(-4)+4}{3}+\frac{1}{2}(-4)\stackrel{?}{=}\frac{1}{4}(-4)-\frac{7}{3}$$ Let $x=-4$.

$$\frac{-4}{3}+(-2)\stackrel{?}{=}-1-\frac{7}{3}$$ Simplify.

$$-\frac{10}{3}=-\frac{10}{3}$$ Simplify.

$$0=0$$ True.

The solution set is {– 4}.

IDENTITIES
CONDITIONAL EQUATIONS
CONTRADICTIONS

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An equation satisfied by *all numbers* that are *meaningful replacements* for the variable is an ***identity***.

$$3(x+1) = 3x + 3$$

An equation that is satisfied by *some numbers*, but *not others*, is a ***conditional equation***.

$$2x = 4$$

An equation that has *no solution* is a ***contradiction***.

$$x = x + 1$$

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Example 3

Identifying Types of Equations

3(a) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

$$-2(x+4)+3x=x-8$$

Solution:

$$-2x-8+3x=x-8$$ Distributive property.

$$x-8=x-8$$ Combine like terms.

$$0=0$$ Subtract $\boldsymbol{x}$ and add $\mathbf{8}$. *(both sides)*

When a *true* statement such as $0=0$ results, the equation is an *identity*, and the solution set is **{all real numbers}**.

Example 3

Identifying Types of Equations

3(b) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

$$5x - 4 = 11$$

Solution:

$$5x = 15$$ Add $\mathbf{4}$ *(both sides)*.

$$x = 3$$ Divide by $\mathbf{5}$ *(both sides)*.

This is a *conditional equation*, and its solution set is **{3}**.

Example 3

Identifying Types of Equations

3(c) Determine whether the equation is an *identity*, a *conditional equation*, or a *contradiction*.

$$3(3x-1)=9x+7$$

Solution: $9x-3=9x+7$ Distributive property.

$-3=7$ Subtract $\mathbf{9x}$ *(both sides).*

When a *false* statement such as $-3=7$ results, the equation is a *contradiction*, and the solution set is the ***empty set*** or ***null set***, symbolized by **{ }** or **Ø** .

IDENTIFYING TYPES of EQUATIONS

If solving a linear equation leads to a *true* statement such as $0 = 0$, the equation is an ***identity***.
Its solution set is **{all real numbers}.**

If solving a linear equation leads to a *single* solution such as $x = 3$, the equation is ***conditional***.
Its solution set consists of a single element.

If solving a linear equation leads to a *false* statement such as $-3 = 7$, the equation is a ***contradiction***.
Its solution set is **{ }** or **Ø** .

SIMPLE INTEREST FORMULA

A formula is an example of a ***linear equation***
(an equation involving letters).
This is the formula for ***simple interest***.

$$I = Prt$$

I is the variable for simple interest

P is the variable for dollars

r is the variable for annual interest rate

t is the variable for years

ACCUMULATED VALUE FORMULA

This formula gives the ***future value***, or ***maturity value***, ***A*** *("the accumulation")* of ***P*** dollars *("the principle")* invested for ***t*** years at an annual simple interest rate ***r***.

$$A = P(1 + rt)$$

A is the variable for future or maturity value

P is the variable for dollars

r is the variable for annual simple interest rate

t is the variable for years

Example 4

Solving for a Specified Variable

4(a) Solve for t :

Goal: Isolate t on one side.

$$I = Prt$$

Solution:

$$\frac{I}{Pr} = \frac{Prt}{Pr}$$

Divide by Pr *(both sides).*

$$\frac{I}{Pr} = t \quad \text{or} \quad t = \frac{I}{Pr}$$

Example 4

Solving for a Specified Variable

4(b) Solve for $\boldsymbol{P}$:

$$A - P = Prt$$

Goal: Isolate $\boldsymbol{P}$ (the specified variable)

Solution:

$$A = P + Prt$$

Transform so that all terms involving $\boldsymbol{P}$ are on one side.

$$A = P(1 + rt)$$

Factor out $\boldsymbol{P}$.

$$\frac{A}{1+rt} = \frac{P(1+rt)}{1+rt}$$

Divide by $\mathbf{1} + \boldsymbol{rt}$. *(both sides)*

$$\frac{A}{1+rt} = P \quad \text{or} \quad P = \frac{A}{1+rt}$$

Example 4

Solving for a Specified Variable

4(c) Solve for x:

Goal: Isolate x

$$3(2x - 5a) + 4b = 4x - 2$$

Solution: $6x - 15a + 4b = 4x - 2$ Distributive property.

Isolate the x-terms on one side. $6x - 4x = 15a - 4b - 2$

Combine like terms. $2x = 15a - 4b - 2$

Divide by **2** *(both sides)*. $x = \dfrac{15a - 4b - 2}{2}$