

LAW of SINES DERIVATION

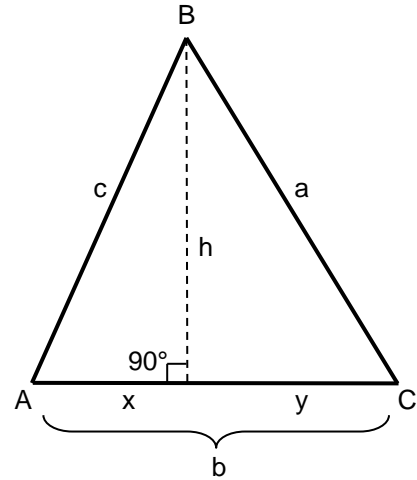
Equation 1 $h = c \cdot \sin A$

Equation 2 $h = a \cdot \sin C$

Equate the right sides of equations 1 and 2 and rearrange...

Equation 3 $\frac{a}{\sin A} = \frac{c}{\sin C}$

Note: $\frac{a}{\sin A} = \frac{b}{\sin B}$ and $\frac{b}{\sin B} = \frac{c}{\sin C}$ are derived similarly.



Law of Sines...

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

LAW of COSINES DERIVATION

Equation 4 $h^2 = a^2 - y^2 = a^2 - (b-x)^2$

Equation 5 $h^2 = c^2 - x^2$

Equate the right sides of equations 4 and 5 and rearrange...

$$a^2 - (b-x)^2 = c^2 - x^2$$

Equation 6 $a^2 = b^2 + c^2 - 2bx$

Equation 7 $x = c \cdot \cos A$

Substitute for x from equation 7 into equation 6...

Equation 8 $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Note: equations for b^2 and c^2 are derived similarly.

Law of Cosines...

alternate versions

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

or

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

or

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

or

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$